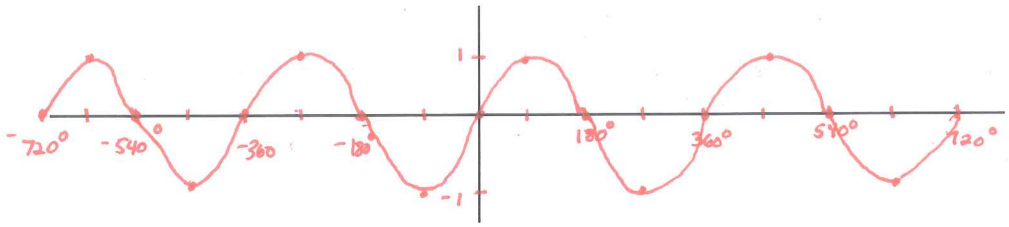
Sec. 7.3 Graphs of Sine and Cosine

The Sine Wave:

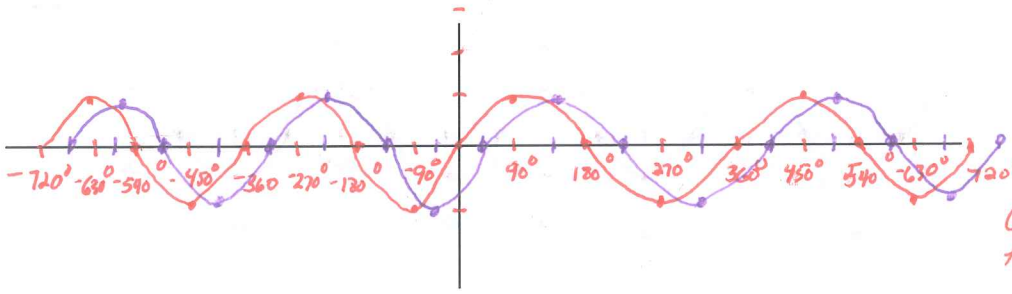


Properties of the Sine Function:

1. The domain is the set of all real numbers.
2. The range consists of real numbers from –1 to 1, inclusive.
3. The sine function is an odd function as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 360° 90°
5. The x-intercepts are … -360°, -180°, 0°, 180°, 360°...
6. The y intercept is 0.
7. The maximum value is 1 and occurs at x = …--270°, 90°, 450°…
8. The minimum value is –1 and occurs at x = …-90°, 270°, 630°…

Transforming Graphs:

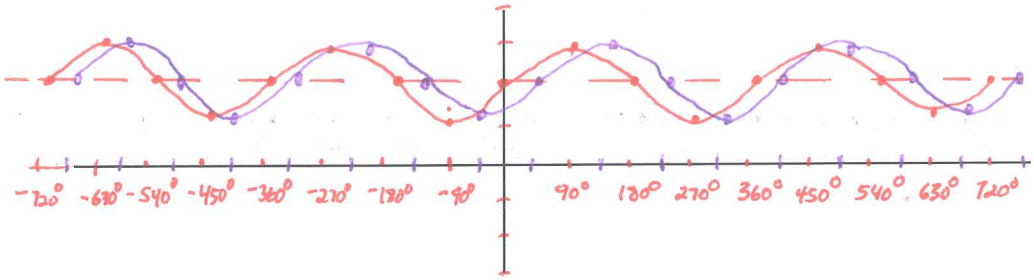
Ex. Graph y= sin x and then graph y = sin (x - 45°), y = sin x + 2 and y = sin (x - 45°) + 2. What do you notice about the graphs? What happens to the x and y intercepts, maximum and minimum values?



y = sin x

y = sin (x - 45°)

Graph shifts 45° to right

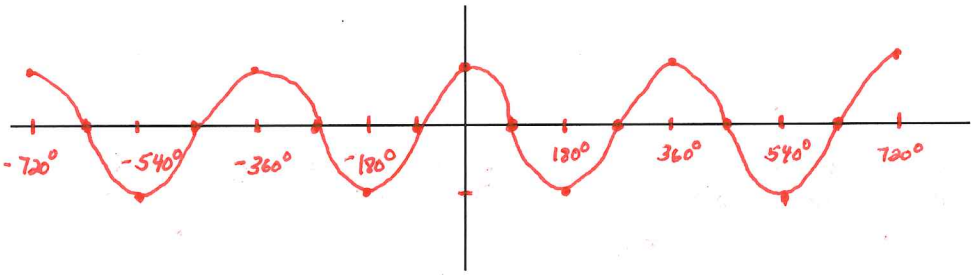


y = sin (x - 45°) + 2

y = sin x + 2

Graph shifts up 2

The Graph of the Cosine Function:

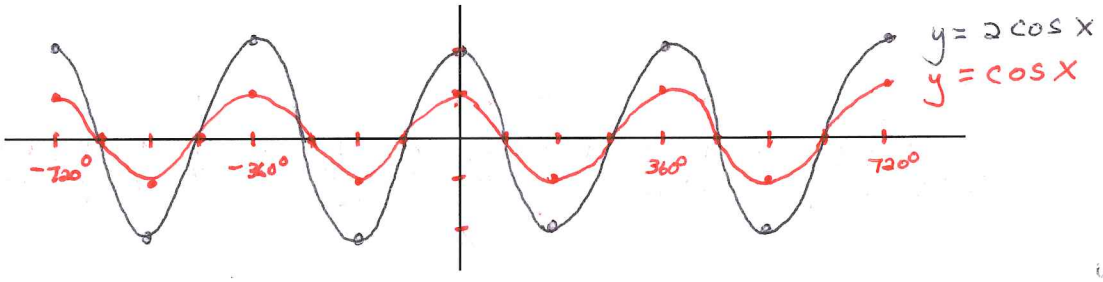


Properties of the Cosine Function:

1. The domain is the set of all real numbers.
2. The range is all real numbers from –1 to 1, inclusive.
3. The cosine function is an even function, with symmetry with respect to the y-axis.
4. The cosine function is periodic with period 360°.
5. The x-intercepts are …-270°, -90°, 90°, 270°, …
6. The y-intercept is 1.
7. The maximum value is 1 and occurs at x = …-360°, 0, 360°, 720°…
8. The minimum value is –1 and occurs at x = …-180°, 180°, 540°, 900°…

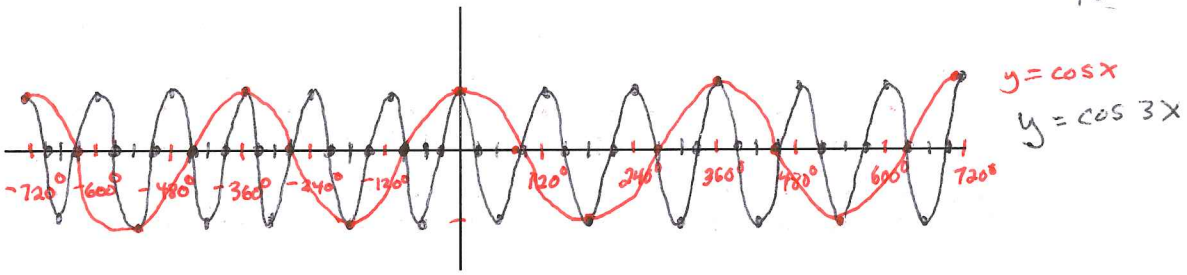
Ex. Compare the graphs of y = cos x and y = 2 cos x and y = cos (3x). What happens to the intercepts and the minimums/maximums? How about period, amplitude and midlines?

$y = 2\cos x$ ← Vertical Stretch SF 2 (Amplitude of 2)
$y = \cos(3x)$ ← Horizontal Compression SF 1/3 (Period = $360 \times \frac{1}{3} = 120°$)



$y = \cos 3x$

Period = $\frac{360}{3} = 120°$



Properties of the sine and cosine functions that are apparent from the graph include:

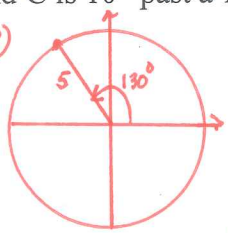
- Domain: All values of θ, since any angle, positive or negative, specifies a point on the unit circle.
- Range: Since values of the sine and cosine are coordinates of points on the unit circle, they lie between −1 and 1. So the range of the sine and cosine are $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$.
- Odd/Even Symmetry: The sine function is odd and the cosine function is even: $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$.
- Period: Both sine and cosine are periodic functions, because the values repeat regularly. The smallest interval over which the function values repeat—here 360°—is called the period. We have

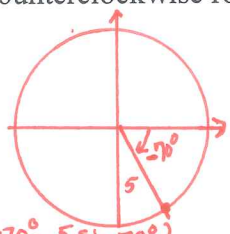
$$\sin(\theta + 360^\circ) = \sin\theta \text{ and } \cos(\theta + 360^\circ) = \cos\theta.$$

The coordinates of the point *P* on the unit circle in the figure are given by $x = \cos\theta$ and $y = \sin\theta$. The coordinates (x, y) of the point Q are given by $x = r\cos\theta$ and $y = r\sin\theta$.

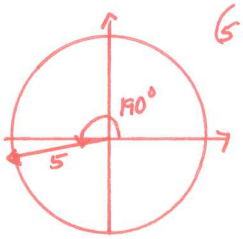
Ex. Find the coordinates of point A, B, and C if the radius of the circle is 5 and A has an angle of 130 degrees counterclockwise rotation, B has an angle of 70° clockwise rotation, and C is 10° past a 180° counterclockwise rotation.

(5cos 130°, 5sin 130°)
(-3.2139, 3.8302)



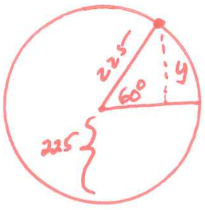


(5cos -70°, 5 sin -70°)
(1.7101, -4.6985)

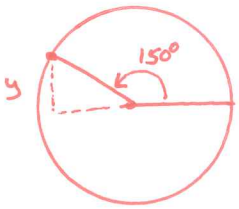


(5cos 190°, 5sin 190°)
(-4.9240, -.8682)

Ex. The Ferris wheel has a radius of 225 feet. Find your height above the ground as a function of the angle θ measured from the 3 o'clock position. What is your height when $\theta = 60^\circ$? when $\theta = 150^\circ$? (note: the center of the ferris wheel is also 225 feet above the ground to begin with!).



$y = 225\sin 60^\circ$
$y = 194.8557$
$+225$
419.8557 ft



$y = 225\sin 150^\circ$
$= 112.5$
$+ 225$
337.5 ft

HW: pg 290-291, #3-27 (m/3) and 29, 31, 32, 34, 36